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Re: Analytical Task

For my analytical task that I decided to research into, I looked at the fatigue strength of our vital gears, both due to bending and contact, to determine their life cycle duration. The first equation I had to used was to calculate the pitch line velocity of the gears in question, given by equation 1.0

$$V = \pi * d * \frac{n}{12}$$

- V = Pitch line velocity [ft/min]
- d = gear diameter [inch]
- n = angular velocity [rpm]

This equation is required to calculate the transmitted power, equation 1.1.

$$W = 33{,}000 * \frac{H}{V}$$

- W = Transmitted power
- H = Required Power [Horsepower]
- V = Pitch line velocity [ft/min]

Transmitted power is needed for the gear bending and contact stress equations, equation 1.2 and 1.3 respectively.

$$\sigma_b = W * K_O * K_V * K_S * \frac{P_d}{F} * K_m * \frac{K_B}{J}$$

- $\sigma_b$ = Gear bending stress
- W = Transmitted power
- $K_O$ = Overload Factor
- $K_V$ = Dynamic Factor
- $K_S$ = Size Factor
- $P_d$ = Diametral Pitch [inch^(-1)]
- $F$ = Force required [lbf]
- $K_m$ = Load distribution Factor
- $K_B$ = Rim thickness Factor
- $J$ = Polar moment of Inertia

$$\sigma_c = C_P * \left(W * K_O * K_V * K_S * \frac{K_m}{d_P * F * I}\right)^{1/2}$$

- $\sigma_c$ = Gear contact stress
- $C_P$ = Elasticity Coefficient [psi^1/2]
- W = Transmitted power
- $K_O$ = Overload Factor
- $K_V$ = Dynamic Factor
- $K_S$ = Size Factor
- $K_m$ = Load distribution Factor
- $d_P$ = Pitch Diameter [inch]
- $F$ = Force required [lbf]
- $I$ = Moment of Inertia

All the intermediate variables in the above two equations are found and tabulated in the tables and figures of the course textbook for Machine Design I [1]. Once an appropriate gear stress is calculated one must calculate the factor of safety, both bending and contact, in order to determine the actual stresses present in the gears. This can be done by using equation 1.4 and 1.5 listed below.

$$S_F = S_t * \frac{Y_N}{\sigma_b * K_T * K_R}$$

- $S_F$ = Bending Factor of Safety
- $S_t$ = Ultimate bending strength
- $Y_N$ = Stress Cycle Factor for Bending Stress
- $\sigma_b$ = Gear bending stress
- $K_T$ = Temperature Factor
- $K_R$ = Reliability Factor

$$S_H = S_c * Z_N * \frac{C_H}{\sigma_c * K_T * K_R}$$

- $S_H$ = Contact Factor of Safety
- $S_c$ = Ultimate contact strength
- $Z_N$ = Stress Cycle Factor for pitting resistance
- $\sigma_c$ = Gear contact stress
- $K_T$ = Temperature Factor
- $K_R$ = Reliability Factor

Again all intermediate variables in the above two equations are found and tabulated in the tables and figures of the course textbook for Machine Design I [1]. Now that the factor of safeties can be calculated we can now calculate the actual bending and contact stresses present in the gears, using equations 1.6 and 1.7 respectively.

$$\sigma_{b,all} = S_t * \frac{Y_N}{S_F * K_T * K_R}$$

- $\sigma_{b,all}$ = Actual bending stress

- $S_t$ = Ultimate bending strength
- $Y_N$ = Stress Cycle Factor for Bending Stress
- $S_F$ = Bending Factor of Safety
- $K_T$ = Temperature Factor
- $K_R$ = Reliability Factor

$$\sigma_{c,all} = S_C * \frac{Z_N}{S_H * K_T * K_R}$$

- $\sigma_{c,all}$ = Actual contact stress
- $S_c$ = Ultimate contact strength
- $Z_N$ = Stress Cycle Factor for pitting resistance
- $S_H$ = Contact Factor of Safety
- $K_T$ = Temperature Factor
- $K_R$ = Reliability Factor

After determining the stresses based on all the materials being the same and gear dimensions of 2.25 in diameter, face width of 2 in, spinning at 6.5 rpm with 21 teeth, it becomes clear that all the internal gears will fail due to bending before they fail due to contact.

These results show that if the team wants a final project that will last for years to come then the material selection is key, they must select materials with a high bending stress limit in order to stay functional. The team has also considered using a type of aluminum alloy, for cost reasons, that does not have a yield limit, meaning that the aluminum material used would be guaranteed to fail after a certain amount of cycles used, roughly 3x10^6 cycles.

**MATLAB Code**

```
%% Input variables
d = 2.65; %Gear diameter [in]
Nt = 22; % Number of teeth
F = 10; % Force [lbf]
P_d = Nt/d; % Diametral Pitch [in^-1]
d_p = pi*d/Nt; % pitch diameter [in]
H = 0.05364; %Power [horsepower]
m_g = 25/21; % pitch diameter ratio
theta = 20; %Pressure angle [degrees]
l = cosd(theta)*sind(theta)/2*(m_g/(m_g+1));
J = pi*d^4/32; % Polar moment
n = 2.5; % Rotational Speed [rpm]
V = pi*d*n/12; % ft/min
W = 33000*H/V; % lbf
K_o = 1; % Overload Factor
Q_v = 5; % quality rating
B = 0.25*(12-Q_v)^(2/3);
A = 50+56*(1-B);
K_v = ((A+sqrt(V))/A)^B; % Dynamic Factor
K_s = 1; %Size factor
C_e = 1;
C_pm = 1;
C_mc = 1;
C_pf = F/10/d_p-0.0375+0.0125*F;
C_ma = 0.127+0.0158*F-(0.000093)*F^2;
K_m = 1+C_mc*(C_pf*C_pm+C_ma*C_e); % Load distribution
factor
K_B = 1; % Rim thickness factor
C_p = 1750; % Elasticty coefficient (psi)^1/2
sig_b1 = W*K_o*K_v*K_s*P_d*K_m*K_B/F/J;
sig_c1 = C_p*sqrt(W*K_o*K_v*K_s*K_m/d_p/F/l);
sigb_lim = 50; % Bending stress limit of material
Y_n = 1.1; % Stress cycle factor for bending stress
K_t = 1; % Temperature factor
K_r = 1; % Reliability factor
sigc_lim = 50; % Contact stress limit of material
Z_n = 1.1; % Stress cycle factor for pitting resistance
C_h = 1; % Hardness Ratio factor for pitting resistance
```

```
S_h = sigc_lim*Z_n*C_h/K_t/K_r/sig_c1; % Pitting factor of
safety
S_f = sigb_lim*Y_n/K_t/K_r/sig_b1; % Bending stress factor
of safety
%% Gear Fatigue Equations
sig_b = sigb_lim*Y_n/S_f/K_t/K_r % Bending stress fatigue
sig_c = sigc_lim*Z_n*C_h/S_h/K_t/K_r % Contact stress
fatigue
```

## References


- [1] – R. Budynas, *Mechanical Engineering Design*. Shigley, 2010